The use of numerical models for ground support systems optimisation: applications, methods and challenges

by Gordon Sweby, Winthrop Professors Phil Dight and Yves Potvin, Australian Centre for Geomechanics

Introduction

Numerical modelling provides an attractive design option for geomechanical engineers as it provides a means to analyse rock mass behaviour and support interaction from engineering 'first-principles', rather than having to rely on precedent, experience and/or empirical methods. This article examines the numerical tools available to engineers, the scenarios in which they are applicable and the critical input parameters required for meaningful analysis.

In order to design a ground support system for an underground mine, (comprising typically rockbolts, mesh and/or fibrecrete), the engineer must first establish the potential failure mechanism, which is dependent on the rock mass and stress conditions. Figure 1 (Potvin 2013) gives an outline of the range of different failure mechanisms which could be encountered. Table 1 lists examples of the range of numerical codes available for ground support design, given a particular mechanism.

Having established an applicable numerical code, the design engineer must determine the input parameters required to simulate the rock mass, loading cycles and ground support systems. These may vary in complexity depending on the code selected, but usually comprise of the following, or subsets thereof:

- Stress state – at the loading stage under consideration.
- Rock mass material model.
- Rock mass fabric.
- Ground support geometrical and mechanical properties.
- Numerical model control parameters.

Case study

To demonstrate the application and variability in outcomes of key input parameters, a simple two-dimensional numerical modelling code, Phase² (Rocscience Inc. 2014), is used as illustration. By examining the rockbolt and liner loads in response to varying key inputs, an appreciation of the variability in potential outcomes can be gained.

A simple example typical of that encountered in Western Australia is shown in Figure 2. Base case input parameters are listed in Table 2.

Material model

The material model selected (Generalised Hoek–Brown) (Hoek et al. 2002) has been adapted in accordance with the methodology described in Diederichs (2007), whereby peak and residual strength parameters are selected such that strain-softening behaviour occurs close to the excavation perimeter whilst under increasing confinement, strain-hardening occurs.

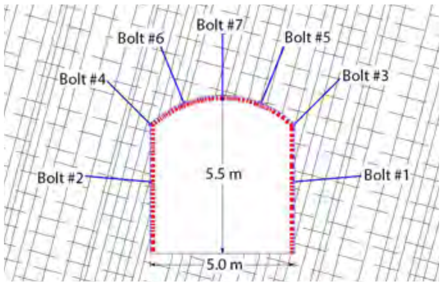

Figure 2 5 x 5.5 m arched tunnel in jointed rock mass, supported by rockbolts (Split-Sets) and liner (shotcrete)

In situ stress

The stress state used in the base case analysis is as follows:

σ_1 – 30 MPa (horizontal).
σ_2 –15 MPa (horizontal, out of plane).
σ_3 – 15 MPa (vertical).

By varying the in situ stress components within realistic bounds (+/- 10%), the model sensitivity in terms of rockbolt and liner load can be determined.

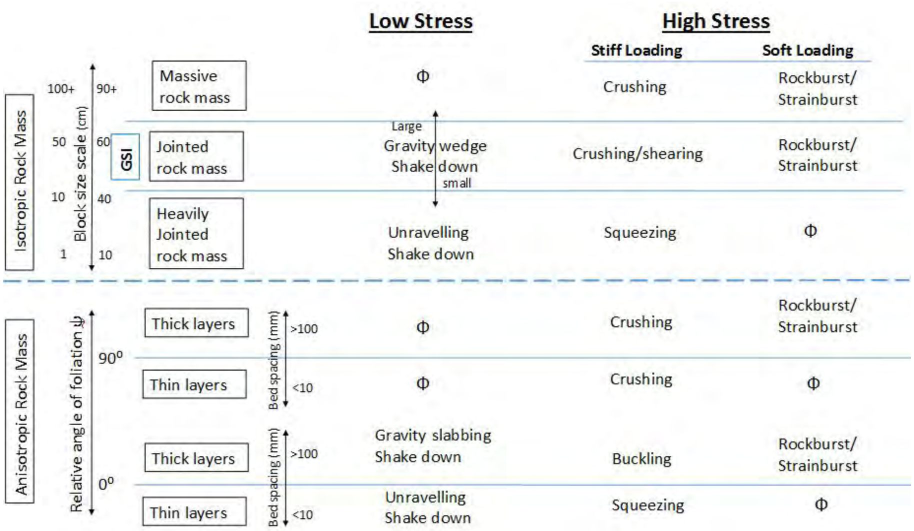

Figure 1 Potential failure mechanisms. The symbol Φ denotes that failure is unlikely

Table 1 Examples of numerical codes applicable per failure mechanism category

		Low Stress	High Stress	
			Stiff Loading	Soft Loading
Isotropic Rock Mass	Massive	N/A	FLAC2D/3D, Phase², ABAQUS	FLAC2D/3D (Dynamic), ABAQUS
	Jointed	UDEC/3DEC, UNWEDGE, Phase², JBLOCK	FLAC2D/3D, Phase², ABAQUS	FLAC2D/3D (Dynamic), ABAQUS
	Heavily Jointed	FLAC2D/3D, Phase², ABAQUS	FLAC2D/3D, Phase², ABAQUS	N/A
Anisotropic Rock Mass	Thick Layers	N/A	UDEC/3DEC, Phase², ABAQUS	FLAC2D/3D (Dynamic), ABAQUS
	Thin Layers	FLAC2D/3D	FLAC2D/3D	N/A

Table 2 Key input parameters

Material Model	Generalised Hoek–Brown
Young's modulus	60 GPa
Poisson's ratio	0.3
Density	2.7 t/m^3
Uniaxial compressive strength	95 MPa
Dilation	0.66
Hoek–Brown m (peak)	1
Hoek–Brown m (residual)	6
Hoek–Brown s (peak)	0.033
Hoek–Brown s (residual)	0.0001
Hoek–Brown a (peak)	0.25
Hoek–Brown a (residual)	0.75
Joint model	Barton–Bandis
Joint compressive strength	90 MPa
Joint roughness coefficient	9
Residual friction angle	28
Joint normal stiffness	600,000 MPa
Joint shear stiffness	60,000 MPa
Rockbolt model	Elastic
Rockbolt modulus	200 GPa
Rockbolt shear stiffness	100 MN/m
Rockbolt bond strength	0.17 MN/m
Liner model	Elastic
Liner modulus	30 GPa
Liner Poisson's ratio	0.2
Liner thickness	0.1 m

Elastic properties

The Young's modulus and Poisson's ratio of the intact rock blocks between the joints were varied as follows:

Young's modulus – 55-65 GPa.

Poisson's ratio – 0.27-0.3.

The sensitivity of rockbolt load is shown in Figure 3. The sidewall bolts (1 and 2) show the most variability, +/- 5 t in each instance.

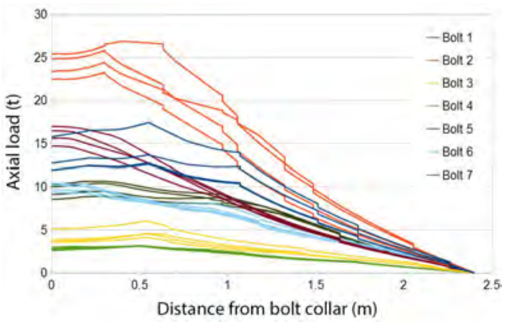

Figure 3 Sensitivity of rockbolt load to varying elastic properties

Intact rock strength

The uniaxial compressive strength of the intact rock blocks varied between 90 and 100 MPa (+/- 5 MPa). With the exception of sidewall bolt 2, the variability is minimal. The variation in bolt 2 is ~6 t.

Joint properties

Joint properties varied as shown in Table 3. Note that in the case of joint shear stiffness, the parameter varied between 60 GPa, being the software developer's recommendation (Rockscience Inc. 2014) and an upper bound, being the value based on elastic equivalence between normal and shear stiffness, i.e. assuming identical Young's modulus and Poisson's ratio for the joint 'infill'.

Table 3 Joint parameter values used in sensitivity analysis

	Upper Bound	Lower Bound
Joint shear stiffness	230 GPa	60 GPa
Joint roughness coefficient (Barton & Choubey 1977)	11	7
Post-peak behaviour	Perfectly plastic	Residual strength

Joint network geometry

The sensitivity of bolt loads on joint network geometry was investigated by re-randomising the base-case, using identical statistical parameters for joint spacing, length and persistence. A variation on the style of jointing was also incorporated by applying a 'cross-jointed' model available in Phase^2.

The impact of varying joint pattern on rockbolt load is shown in Figures 4 and 5, for bolts 2 and 5. The range varies from ~15 t in the case of bolt 2, to ~6 t for bolt 5.

Combined sensitivity

Combining all the analysed parameters into a single sensitivity plot enables a preliminary assessment of the overall range in expected outcomes, in terms of rockbolt load.

The results for rockbolts 2 and 5 (being the bolts showing most variability in the previous analyses) are shown in Figures 6 and 7. The maximum variations in axial load for bolts 2 and 5 are ~12 and 5 t respectively.

Similarly, the combined sensitivity of the liner radial and axial loads is shown in Figure 8. The radial (shear) loads vary between +25 and -25 t, whilst the axial loads range from 0 to 400 t.

Design criteria

In order to assess a ground support design for acceptability, the engineer must decide on some acceptance criteria, for example:

- Factor of Safety.
- Probability of failure.
- Residual support capacity.

This can be problematic when using numerical models, due to the complexity in the failure mechanisms. Global capacity versus demand approaches can be meaningless in instances where bolts in the pattern are loaded differently, depending on their position around the perimeter: which bolt is selected for the capacity versus demand calculation?

Comparisons between different scenarios are often helpful in making broad assessments of ground support effectiveness, but are not really sufficient for engineering design purposes.

Discussion

For the simple example of rockbolt and liner loading chosen, it has been demonstrated that a significant range in potential outcomes are possible, based on the natural variability (known and/or unknown) of the rock materials and discontinuities.

The challenge facing the geotechnical engineer is basing a design on this level of uncertainty. Ideally a full probabilistic analysis or response surface analysis (RSM) would be carried out, varying all of the key parameters and generating a sufficient number of outcomes to be statistically justifiable. However, the time and computational effort required would

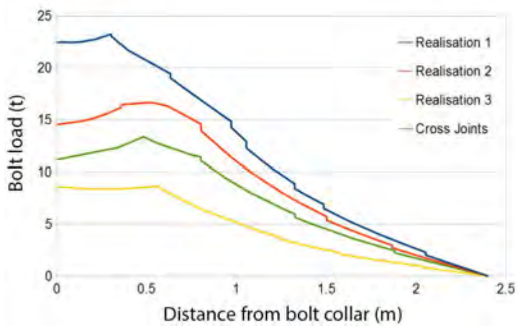

Figure 4 Sensitivity of rockbolt (2) load to variation in joint network geometry

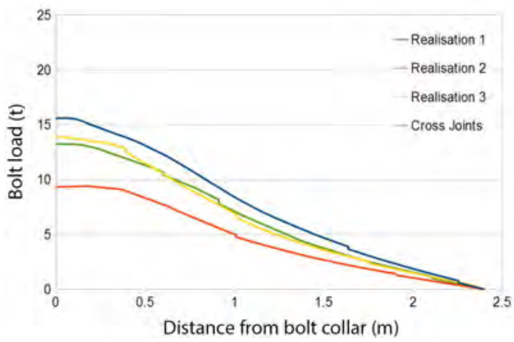

Figure 5 Sensitivity of rockbolt (5) load to variation in joint network geometry

put this approach beyond the means of practitioners.

Simple sensitivity analyses do provide the means for judgment calls, e.g:

- Design for the worst-possible scenario. Base the rockbolt capacity on the maximum load profile generated; or
- Design for an average condition (most probable outcome).

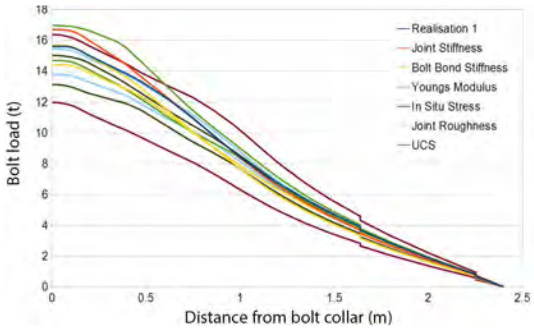

Figure 6 *Combined sensitivity for all parameters analysed, but excluding variations in joint geometry – rockbolt 4*

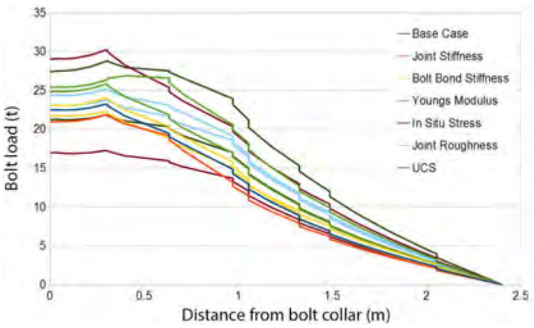

Figure 7 *Combined sensitivity for all parameters analysed, but excluding variations in joint geometry – rockbolt 2*

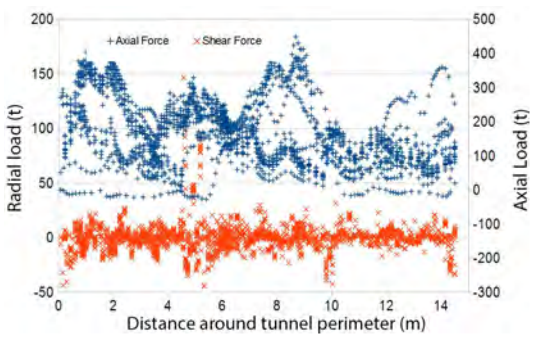

Figure 8 *Combined sensitivity of liner axial and radial loads, for all parameters analysed*

Table 4 *Relative importance of the parameters analysed (listed in order of sensitivity)*

	Parameter	Variation	Sensitivity (t)
1	Uniaxial compressive strength	+/- 5%	7.0
2	In situ stress	+/- 10%	13.0
3	Young's modulus	+/- 8%	2.7
4	Joint roughness coefficient	+/- 22%	1.4
5	Rockbolt bond stiffness	+/- 20%	1.8
6	Poisson's ratio	+/- 11%	1.0

When embarking on ground support design using numerical models, it is clear that appropriate effort should be directed towards reducing uncertainty in the input parameters. For the example chosen and parameters analysed, their order of importance is as listed in Table 4.

However, the impact of known unknowns such as joint network and opening geometry may overshadow the importance of other parameters. Thus it is unavoidable that an appropriate number of joint network simulations be carried out.

Conclusions

The challenges in optimising ground support by the application of numerical models are significant, due to the natural variability in the input parameters which greatly affect the outcomes.

Furthermore, numerical models alone do not provide the engineer with an absolute means of assessing the appropriateness of ground support designs in terms of safety and serviceability.

The introduction of probabilistic methods, in combination with numerical modelling, may provide a means to incorporate natural variability in modelling parameters into the design, whilst simultaneously allowing the engineer to make rational decisions on excavation serviceability, based on industry accepted criteria.

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The use of numerical models for Ground Support Systems Optimisation: Applications, Methods and Challenges

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ABSTRACT: One of the main challenges facing geomechanics practitioners is the technical and financial justification of ground support systems at their mines, in order to satisfy management and legislative requirements. Numerical modelling provides an attractive option for support design, due to the highly technical and 'engineered' solutions they provide. This paper examines the various numerical methods available to practitioners, reviews the types of geomechanical environments in which they are applicable and comments on potential pitfalls, risks and variability in outcomes. A case study of the application ground support design in a jointed hard-rock environment, using DFN methodology, is used as illustration.

1. INTRODUCTION

Numerical modelling provides an attractive design option for geomechanical engineers as it provides a means to analyse rock mass behaviour and support interaction from engineering 'first-principles', rather than having to rely on precedent, experience and/or empirical methods. This paper examines the numerical tools available to engineers, the scenarios in which they are applicable and the critical input parameters required for meaningful analysis.

In order to design a ground support system for an underground mining scenario (comprising typically rockbolts, mesh and/or fibrecrete), the engineer must first establish the potential failure mechanism, which is dependent on the rockmass and stress conditions. Figure 1 [1] gives an outline of the range of different failure mechanisms which could be encountered in an underground mining scenario. Table 1 lists examples of the range of numerical codes available for ground support design given a particular mechanism.

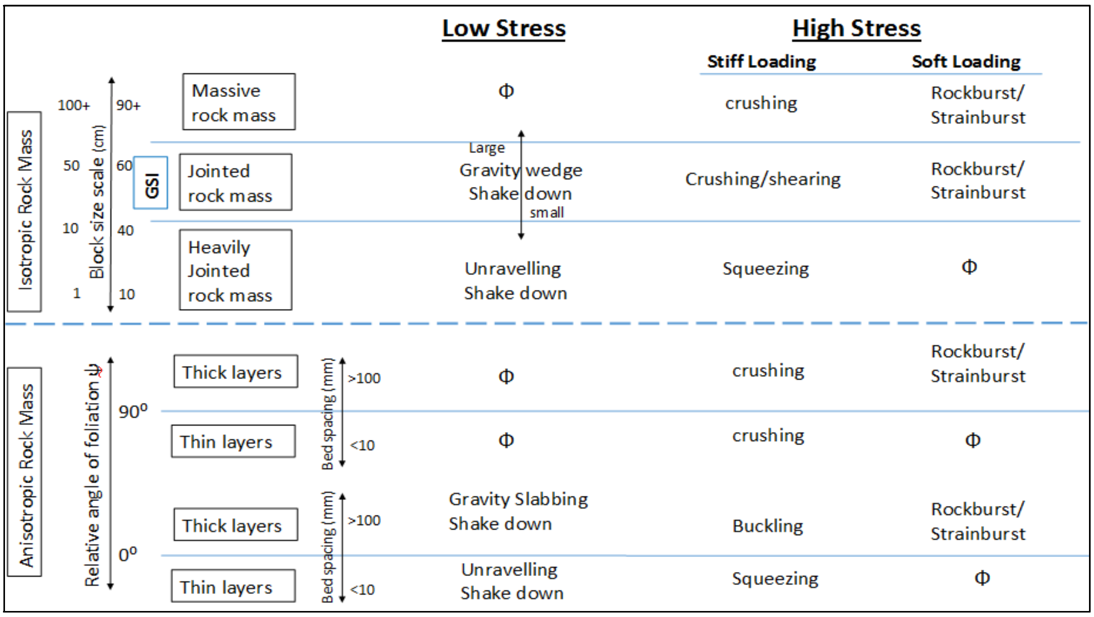

Figure 1. Potential failure mechanisms. The symbol Φ denotes that failure is unlikely.

Table 1. Examples of numerical codes applicable per failure mechanism category

		Low Stress	High Stress	
			Stiff Loading	**Soft Loading**
Isotropic Rockmass	**Massive**	N/A	FLAC2D/3D, PHASE2, ABAQUS	FLAC2D/3D (Dynamic), ABAQUS
	Jointed	UDEC/3DEC, UNWEDGE, PHASE2, JBLOCK	FLAC2D/3D, PHASE2, ABAQUS	FLAC2D/3D (Dynamic), ABAQUS
	Heavily Jointed	FLAC2D/3D, PHASE2, ABAQUS	FLAC2D/3D, PHASE2, ABAQUS	N/A
Anisotropic Rockmass	**Thick Layers**	N/A	UDEC/3DEC, PHASE2, ABAQUS	FLAC2D/3D (Dynamic), ABAQUS
	Thin Layers	FLAC2D/3D	FLAC2D/3D	N/A

Having established an applicable numerical code, the design engineer must determine the input parameters required to simulate the rockmass, loading cycles and ground support systems. These may vary in complexity depending on the code selected, but usually comprises the following, or subsets thereof;

- Stress State – at the loading stage under consideration
- Rockmass material model
- Rockmass fabric
- Ground Support geometrical and mechanical properties
- Numerical model control parameters

2. CASE STUDY

To demonstrate the application and variability in outcomes, of key input parameters, a simple two-dimensional numerical modelling code: Phase2 [2] will be used as illustration. By examining the rockbolt and liner loads in response to varying key inputs, an appreciation of the variability in potential outcomes can be gained.

A simple example typical of that encountered in Western Australia is shown in Figure 2. Base case input parameters are listed in Table 2.

3. Material Model

The material model selected (Generalised Hoek-Brown)[3] has been adapted in accordance with the methodology described in Diederichs [4],whereby peak and residual strength parameters are selected such that strain-softening behaviour occurs close to the excavation perimeter whilst under increasing confinement (i.e. further from the excavation perimeter), strain-hardening occurs.

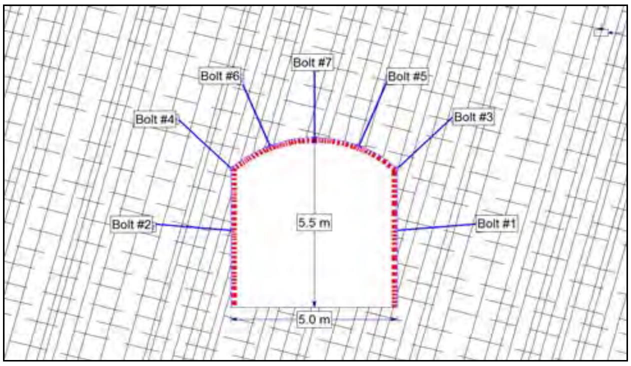

Figure 2. 5 x 5.5m arched tunnel in jointed rockmass, supported by rockbolts (Split-Sets) and liner (shotcrete).

The parameters used are as given in Table 2.

4. In situ Stress

The stress state used in the base case analysis is as follows;

σ1	-	30MPa (horizontal)
σ2	-	15MPa (horizontal, out of plane)
σ3	-	15MPa (vertical)

By varying the in situ stress components within realistic bounds (+/-10%) the model sensitivity in terms of rockbolt and liner load can be determined, as shown in Figure 3. For each bolt, an upper and lower bound is shown.

The largest range in variability occurs in sidewall bolt 2: ~12 tonnes. Bolt 5 in the back of the excavation indicates a variation of ~5 tonnes.

Table 2. Key Input Parameters

Material Model	:	Generalised Hoek-Brown
Young's Modulus	:	60GPa
Poisson's ratio	:	0.3
Density	:	2.7 t/m3
Uniaxial Compressive Strength	:	95MPa
Dilation	:	0.66
Hoek-Brown m (peak)	:	1
Hoek-Brown m (residual)	:	6
Hoek-Brown s (peak)	:	0.033
Hoek-Brown s (residual)	:	0.0001
Hoek-Brown a (peak)	:	0.25
Hoek-Brown a (residual)	:	0.75
Joint Model	**:**	**Barton-Bandis**
Joint Compressive Strength	:	90MPa
Joint Roughness Coefficient	:	9
Residual Friction Angle	:	28
Joint Normal Stiffness	:	600000MPa
Joint Shear Stiffness	:	60000MPa
Rockbolt Model	**:**	**Elastic**
Rockbolt Modulus	:	200GPa
Rockbolt Shear Stiffness	:	100MN/m
Rockbolt Bond Strength	:	0.17MN/m
Liner Model	**:**	**Elastic**
Liner Modulus	:	30GPa
Liner Poisson's ratio	:	0.2
Liner thickness	:	0.1m

5. *Elastic Properties*

The Young's Modulus and Poisson's ratio of the intact rock blocks between the joints were varied as follows:

Young's Modulus - 55 – 65 GPa
Poisson's Ratio - 0.27 – 0.3

The sensitivity of rockbolt load is shown in Figure 4. The sidewall bolts (1 and 2) show the most variability, +/- 5 tonnes in each instance.

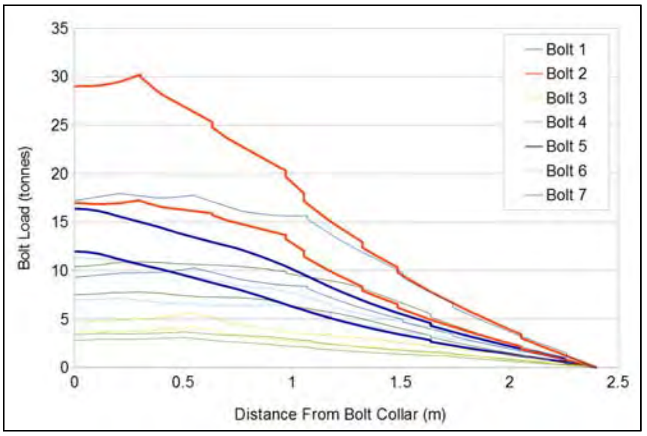

Figure 3. Sensitivity of rockbolt load to varying in situ stress condition.

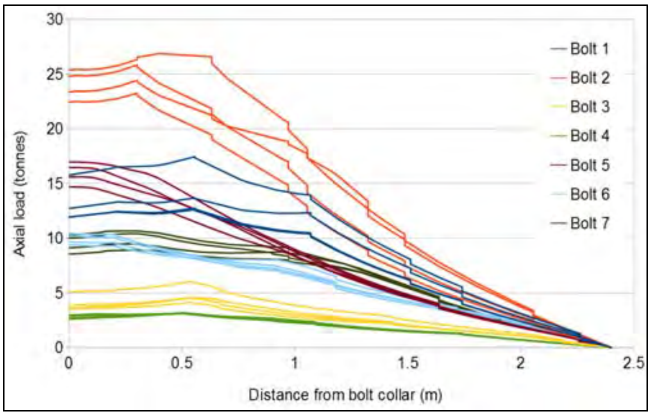

Figure 4. Sensitivity of rockbolt load to varying elastic properties.

6. *Intact Rock Strength*

The uniaxial compressive strength of the intact rock blocks was varied between 90 and 100 MPa (+/- 5MPa) and the impact on rockbolt load is shown in Figure 5. With the exception of sidewall bolt 2, the variability is minimal. The variation in bolt 2 is ~6 tonnes.

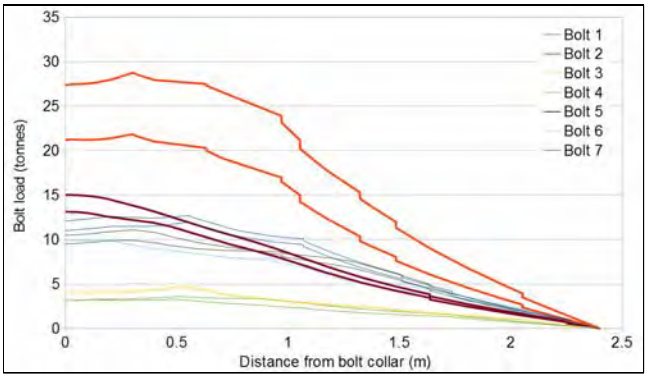

Figure 5. Sensitivity of rockbolt load to uniaxial compressive strength.

7. *Joint Properties*

Joint properties were varied as shown in Table 3. Note that in the case of joint shear stiffness, the parameter was varied between 60GPa, being the software developer's recommendation [2] and an upper bound, being the value based on elastic equivalence between normal and shear stiffness (i.e. assuming identical Young's modulus and poisson's ratio for the joint 'infill')

Table 3. Joint parameter values used in sensitivity analysis

	Upper bound	Lower Bound
Joint Shear Stiffness	230 GPa	60 GPa
Joint Roughness Coefficient [5]	11	7
Post-Peak Behaviour	Perfectly Plastic	Residual Strength

With reference to Figure 6, bolts 2 (~4 tonnes) and 5 (~3 tonnes) show the greatest sensitivity to variation in the given inputs.

8. *Joint Network Geometry*

The sensitivity of bolt loads on joint network geometry was investigated by re-randomising the base-case, using identical statistical parameters for joint spacing, length and persistence. A variation on the style of jointing was also incorporated by applying a 'cross-jointed' model available in Phase2.

The joint patterns generated are shown in Figure 7. The impact of varying joint pattern on rockbolt load is shown in Figures 8 and 9, for bolts 2 and 5. The range varies from ~15 tonnes in the case of bolt 2, to ~6 tonnes for bolt 5.

9. *Combined Sensitivity*

Combining all the analysed parameters into a single sensitivity plot enables a preliminary assessment of the overall range in expected outcomes, in terms of rockbolt load (Note: this is not a probabilistic analysis).

The results for rockbolts 2 and 5 (being the bolts showing most variability in the previous analyses) are shown in Figures 10 and 11 respectively. The maximum variations in axial load for bolts 2 and 5 are ~12 tonne and 5 tonne respectively.

(Note: the variations due to changes in joint network geometry are not included in these figures).

Similarly, the combined sensitivity of the liner radial and axial loads is shown in Figure 12. The radial (shear) loads vary between +25 tonne and -25 tonne, whilst the axial loads range from 0 to 400 tonne.

10. *Design Criteria*

In order to assess a ground support design for acceptability, the engineer must decide on some acceptance criteria, for example;

- Factor of safety
- Probability of failure
- Residual support capacity

This can be problematic when using numerical models, due to the complexity in the failure mechanisms. Global capacity vs demand approaches can be meaningless in instances where, as in this example, bolts in the pattern are loaded differently depending on their position around the perimeter: which bolt is selected for the capacity v demand calculation?

Comparisons between different scenarios is often helpful in making broad assessments of ground support effectiveness but are not really sufficient for engineering design purposes. For example, in Figures 13 and 14 the yielded zones and displacements around the tunnel are compared, for an unsupported vs supported case. Less yield and displacement is clearly evident in the supported case, however the engineer still needs to make an assessment of whether the supported case is actually adequate to provide a serviceable excavation.

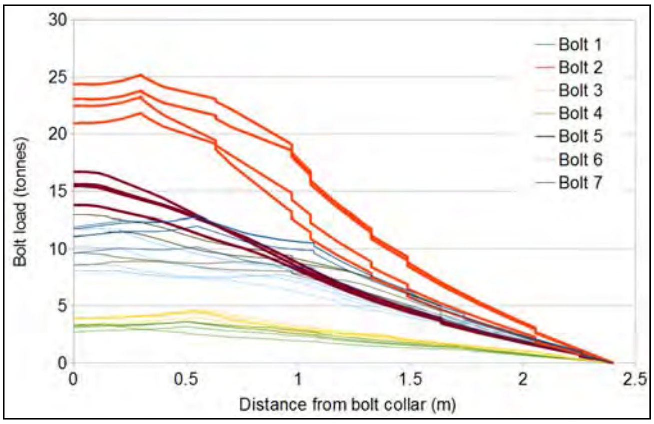

Figure 6. Sensitivity of rockbolt load to joint properties (shear stiffness, roughness and residual strength).

Similarly a comparison between a bolts-only case and a liner-only case (Figures 15 and 16) can provide insight as to which option is more effective (in this case, the liner) but cannot provide the engineer with quantitative guidance on excavation serviceability.

11. DISCUSSION

For the simple example of rockbolt and liner loading chosen, it has been demonstrated that a significant range in potential outcomes are possible, based on the natural variability (known and/or unknown) of the rock materials and discontinuities.

The challenge facing the geotechnical engineer is basing a design on this level of uncertainty. Ideally a full probabilistic analysis or response surface analysis (RSM) would be carried out, varying all of the key parameters and generating a sufficient number of outcomes to be statistically justifiable. However, the time and computational effort required would put this approach beyond the means of practitioners.

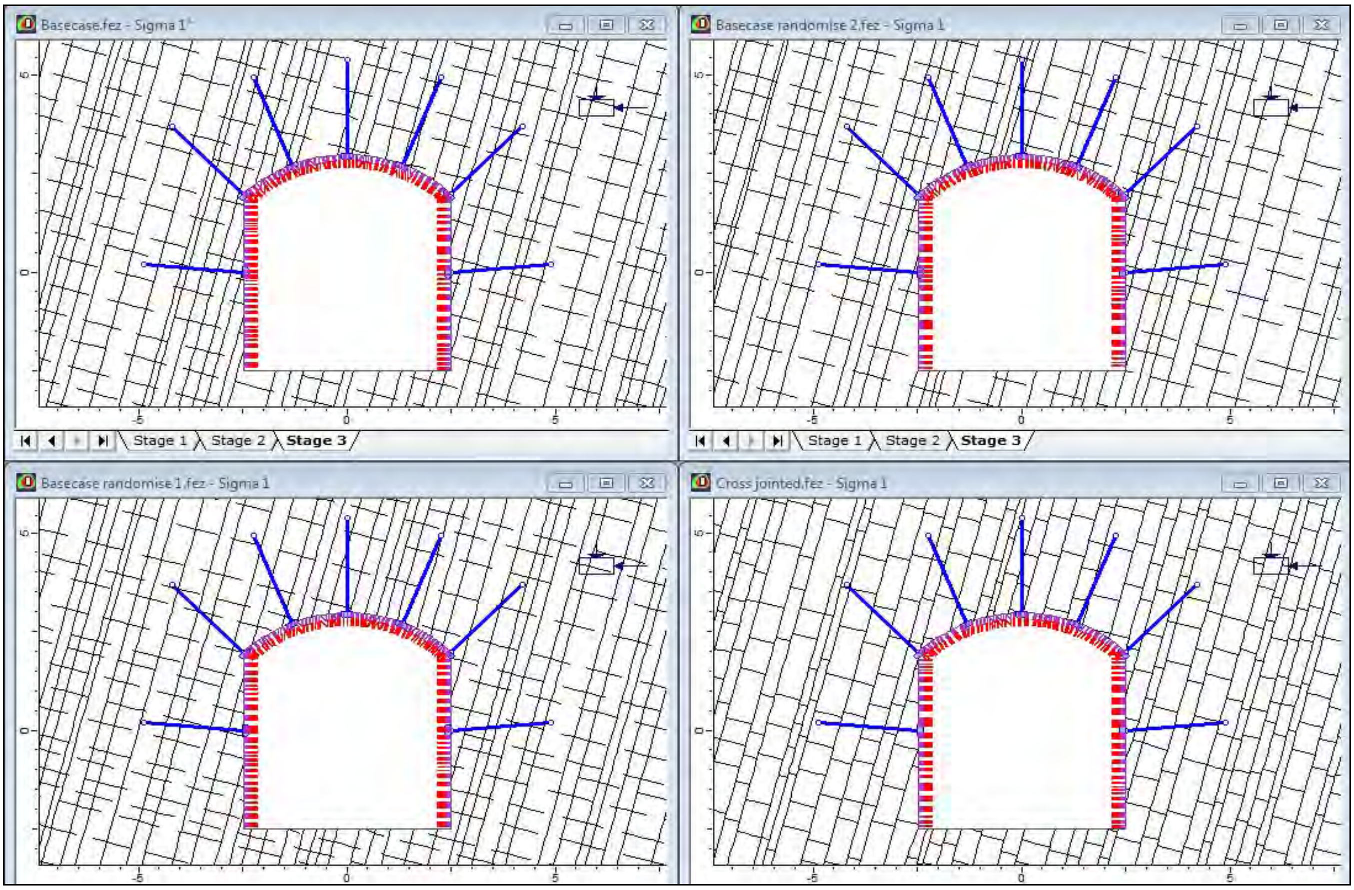

Figure 7. Variations in joint pattern produced by re-randomising statistical distributions of key parameters (spacing, persistence and length). A cross-jointed model is also shown.

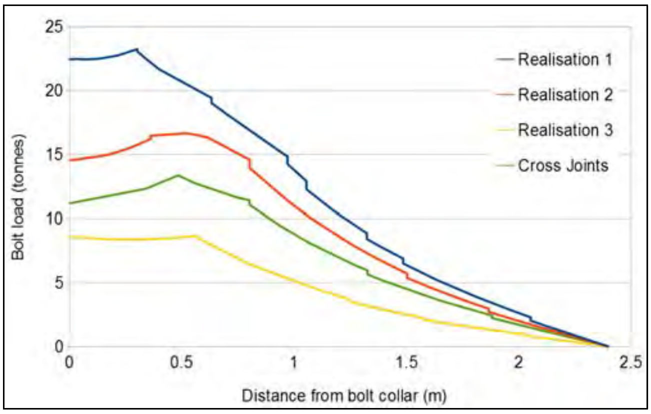

Figure 8: Sensitivity of rockbolt (2) load to variation in joint network geometry.

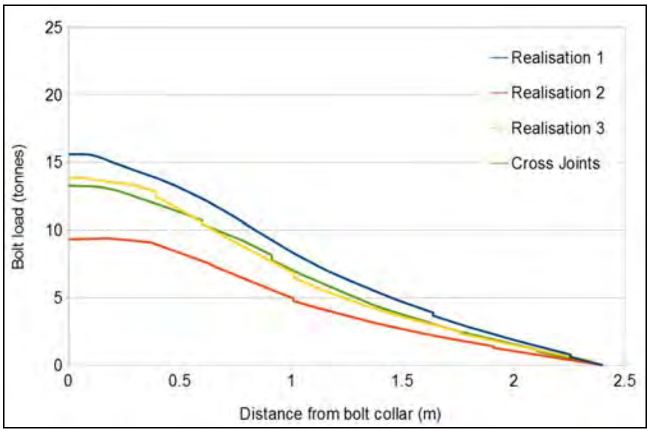

Figure 9: Sensitivity of rockbolt (5) load to variation in joint network geometry.

Simple sensitivity analyses such as demonstrated above however, do provide the means for judgment calls, e.g;

- Design for the 'worst-possible' scenario: base the rockbolt capacity on the maximum load profile generated, or;
- Design for an 'average' condition (most probable outcome).

When embarking on ground support design using numerical models, it is clear that appropriate effort should be directed towards reducing uncertainty in the input parameters. For the example chosen and parameters analysed, their order of importance is as listed in Table 4.

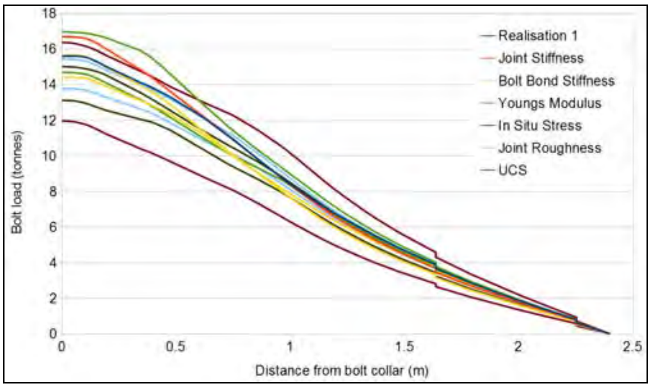

Figure 10. Combined sensitivity for all parameters analysed (but excluding variations in joint geometry) – rockbolt 4.

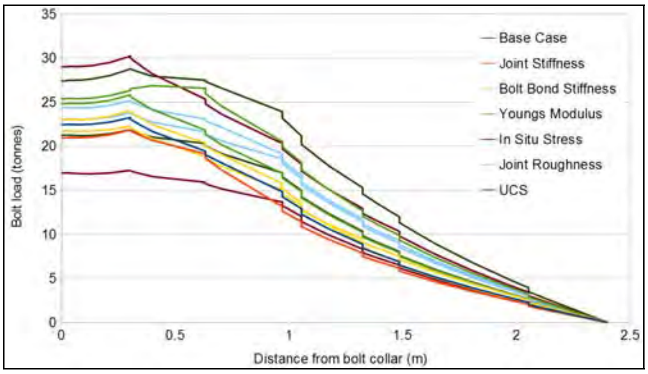

Figure 11. Combined sensitivity for all parameters analysed (but excluding variations in joint geometry) – rockbolt 2.

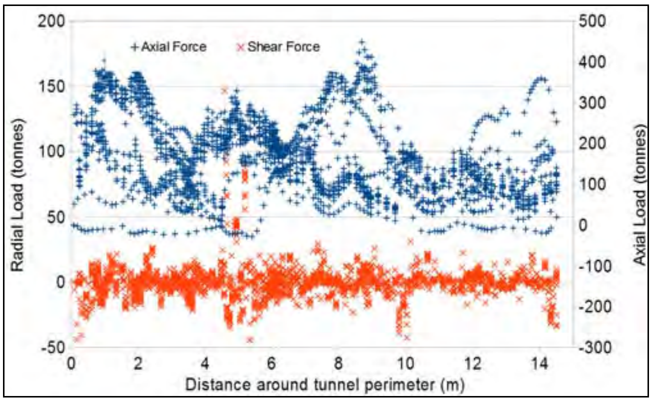

Figure 12. Combined sensitivity of liner axial and radial loads, for all parameters analysed.

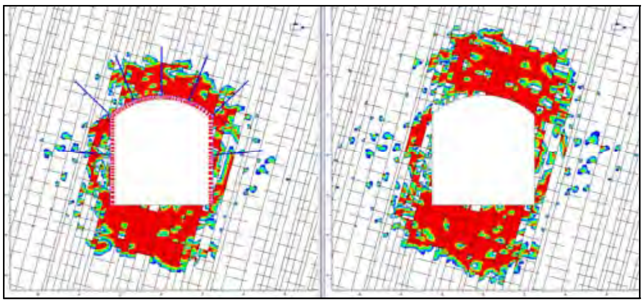
Figure 13. Comparison between the base case (supported) and an unsupported case (decrease in size of the yielded zone).

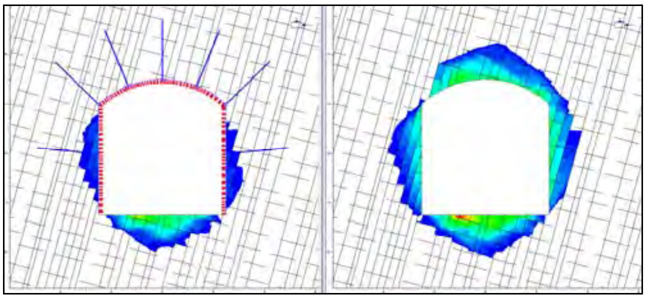
Figure 14. Comparison between the base case (supported) and an unsupported case, showing a decrease in the displacements surrounding the tunnel.

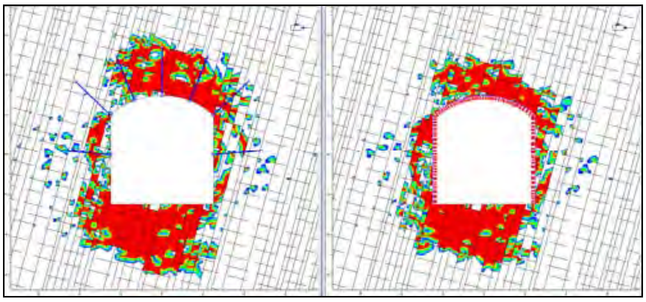
Figure 15, Bolts only vs liner only - yielded zones.

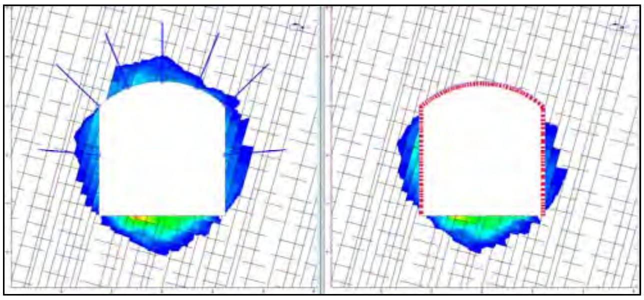
Figure 16. Bolts only vs liner only – displacement.

Table 4: Relative importance of the parameters analysed (listed in order of sensitivity)

Parameter	Variation	Sensitivity (tonnes)
1. Uniaxial Compressive Strength	+/- 5%	7.0
2. In Situ Stress	+/- 10%	13.0
3. Young's Modulus	+/- 8%	2.7
4. Joint Roughness Coefficient	+/- 22%	1.4
5. Rockbolt Bond Stiffness	+/- 20%	1.8
6. Poisson's ratio	+/- 11%	1.0

However the impact of “known unknowns” such as joint network and opening geometry may overshadow the importance of other parameters. Thus it is unavoidable that an appropriate number of joint network simulations be carried out.

12. CONCLUSIONS

The challenges in optimising ground support by the application of numerical models are significant, due to the natural variability in the input parameters which greatly affect the outcomes.

Furthermore, numerical models alone do not provide the engineer with an absolute means of assessing the appropriateness of ground support designs in terms of safety and serviceability.

The introduction of probabilistic methods, in combination with numerical modelling, may provide a means to incorporate natural variability in modelling parameters into the design, whilst simultaneously allowing the engineer to make rational decisions on excavation serviceability, based on industry accepted criteria.

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